Solutions

Math 1A Worksheet: Extrema

Name: _____

April 2, 2025

1. Let $f(x) = x^3 + 6x^2 - 15x$. Find all local maximums and minimums. Is there any global maximums or minimums?

$f'(x) = 3x^2 + 12x - 15$

$\frac{-12 \pm \sqrt{144 + 180}}{6} = -2 \pm \frac{\sqrt{324}}{6} = -2 \pm 3$

crit. pt. $-5, 1$

$f''(x) = 6x + 12$

$f''(-5) < 0$ max $\quad f''(1) > 0$ min.

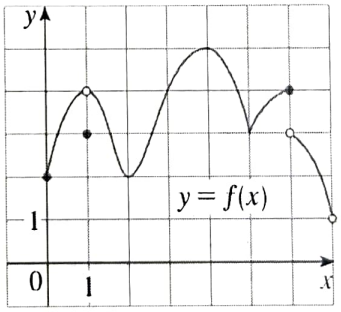
No

2. Find all local and global maximums and minimums for the function $f(x) = \sin(x)$ in the interval $[0, 4\pi]$.

$\pi/2, 5\pi/2$ max

$3\pi/2, 7\pi/2$ min

3. Consider the function $y = f(x)$ shown in the graph below. Find all local and absolute minimums and maximums, if exist.



No global min

$x = 2, 5$ local min

$x = 4, 6$ local max

4. Suppose $f(x) = 5 + 54x - 2x^3$. When is the function increasing/decreasing, concave up/concave down?

$f'(x) = 54 - 6x^2 = 6(9 - x^2)$

Dec. $(-\infty, -3) \cup (3, \infty)$

Inc. $(-3, 3)$

$f''(x) = -12x$

Concave up $(-\infty, 0)$

Down $(0, \infty)$

5. Use a graph to estimate the critical numbers of $f(x) = |1 + 5x - x^3|$.

6. Show that a cubic function $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are constants, can have two, one, or no critical number(s). Give examples and sketches to illustrate the three possibilities.

$f'(x) =$

two $f(x) = (x-1)(x)(x+1)$ ⇝

one ⇝ $f(x) = x^2$

None ⇝ $f(x) = x$

7. Prove that the function

$$f(x) = x^{101} + x^{51} + x + 1$$

has neither a local minimum or a local maximum.

$f'(x) = 101\,x^{100} + 51\,x^{50} + 1 > 0$ (never $= 0$)